Rolling Resistance Measurement and Model Development

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Abstract: There is an increased focus worldwide on understanding and modeling rolling resistance because reducing the rolling resistance by just a few percent will lead to substantial energy savings. This paper reviews the state of the art of rolling resistance research, focusing on measuring techniques, surface and texture modeling, contact models, tire models, and macro-modeling of rolling resistance. **DOI: 10.1061/(ASCE)TE.1943-5436.0000673.**

Introduction

The total annual emission of CO_2 in the United States exceeds 7 billion t of which the transport sector's share is 29% (U.S. Department of Transportation 2010). Consequently, a reduction of rolling resistance will lead to substantial energy savings and CO_2 emission reductions. Although the European Union has been able to reduce total greenhouse gas emissions by approximately 5% between 1990 and 2006, CO_2 emissions from road transport in the same period increased by 26% and now constitute 12% of total CO_2 emissions in the European Union (Schmidt and Dyre 2012), so rolling resistance reductions are also important here.

Fuel consumption, and hence CO_2 emission in road transport, depends on a number of factors that relate to the vehicles, the quality of the road, and their interaction. Low rolling resistance tires have been available from the tire industry since 1993, and every second tire sold today is a low rolling resistance tire.

Rolling resistance related to the road surface is responsible for about 20% of the CO_2 emitted by a passenger car driving at 100 km/h (Haider et al. 2011). This paper focuses on the role of the road surface on rolling resistance.

In order to overcome the resistance, vehicles consume fuel. The resistance can be categorized as follows (Sandberg et al. 2011b):

- Rolling resistance.
- Air resistance.
- Inertial resistance.
- Gradient resistance.
- Side force resistance.
- Transmission loss.
- Losses from the use of auxiliary equipment.
- Engine friction.

The rolling resistance is defined as the energy loss per distance traveled by the vehicle due to nonelastic deformations of the tires and losses in the wheel suspension system. Energy dissipation in asphalt pavement structures also contributes to the rolling resistance, but studies [e.g., Pouget et al. (2012)] show this to be of minor importance. Because energy (measured in Joules) divided by distance (measured in meters) has the unit of force (measured in Newtons), the rolling resistance coefficient, defined as the energy loss per tire per distance travelled divided by the normal force on the tire, is dimensionless.

The emphasis in this paper is on the literature from the 1980s to the present day. First, the different ways of measuring rolling resistance are reviewed. Then the literature on surface roughness and texture modeling is considered. The interaction between road surface and tire is considered in the next two sections, and finally there is a section on macro-modeling of rolling resistance. Due to the wide diversity of areas connected with rolling resistance measuring and modeling, this paper will not go into specifics nor engage in in-depth critiques of the presented material; the aim is primarily to provide an overview of the references in the area and uncover common threads in the research field.

Rolling Resistance Measuring Techniques

Measurements of rolling resistance date back several centuries and originated with the military's interest in reducing the horsepower for the traction of canons (Luchini 1983). Great scientists such as Coulomb and Reynolds contributed to the field. Only in the last 50 years, however, has a systematic treatment been attempted with the aim of establishing standards for the measurement of rolling resistance. Recently, Sandberg et al. (2011b) classified the rolling resistance measuring techniques into:

- Drum tests of tires;
- Trailer methods;
- Coast-down methods; and
- Fuel consumption methods.

The drum test is ideal for testing of tires in the laboratory. Trailer methods add the variation of the road surface to the testing and also monitor some of the transmission loss [see the discussion in Section 3 of the Models for rolling resistance In Road Infrastructure Asset Management systems (MIRIAM) project report Sandberg et al. (2011b)]. Coastdown methods include still further properties

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of the car, while fuel consumption methods measure all the energy losses experienced by the car.

Drum Tests of Tires

Rolling resistance of tires can be measured by the drum test, which, as the name suggests, is performed by holding the test tire up against a drum and applying a load to the tire. By rotating the drum and measuring the resistance the tire exerts on the rotation of the drum, the rolling resistance force of the tire can be deduced. The advantage of this setup is the exclusion of various factors that influence the rolling resistance of a tire. Before 1975 the experiment protocol varied with different studies, e.g., different loads on the test tire, different inflation pressure and drum setup. This was standardized in 1975 when the Society of Automotive Engineers (SAE) formed a committee to standardize the drum testing procedure (Luchini 1983). Today several standards have been published by the International Organization for Standardization (ISO) and SAE [SAE J2452 (SAE 1999), SAE J1269 (SAE 2006), ISO 28580 (ISO 2009), ISO 18164 (ISO 2005)] and a physical and mathematical justification for ISO 18164 has been proposed by Hublau and Barillier (2008). For an overview of the different standards the reader is referred to Gent and Walter (2006, Chapter 12) and Sandberg et al. (2011b). Several aspects of the drum testing procedure have been investigated, such as interlaboratory correlation (Clark and Schuring 1978; O'Neal et al. 1982) and curvature correction (Clark 1976; Luchini 1982) when converting drum results to a flat surface, although the latter was recently disputed by Freudenmann et al. (2009).

Besides direct rolling resistance measurement, drum testing has been used for several other related purposes such as surface texture testing (Luchini and Simonelli 1983), warm-up effects (Warholic 1983), wheel cornering effects (Keefe and Koralek 1983), prediction of cavity air temperature (Kenny 1983), and prediction of transient rolling resistance (Luchini and Popio 2007; Mars and Luchini 1999).

Trailer Methods

The trailer method uses a trailer with one or more test wheels being towed by a vehicle while the test wheels' resistance to rolling is measured by force transducers. The trailer method has been in development since the 1980s up until today and has been documented in Sandberg et al. (2011b). In the 1980s the Belgian Road Research Centre (BRRC) designed a trailer to assess rolling resistance, and in 1990 data produced with the trailer were published and correlated with road profile spectra (Descornet 1990). The trailer was improved in 2009 (Sandberg et al. 2011b). Since then, the Technical University of Gdansk (TUG), the Federal Highway Research Institute of Germany (BASt), and Helsinki University of Technology (HUT) have developed trailers, the latter with limited success, though, according to Leinonen and Juhala (2006). The TUG trailer depicted in Fig. 1(a) has been described in the literature (Wozniak et al. 2011a, b). BRRC, BASt, and TUG trailers are used today in various projects dealing with rolling resistance and road asset management, see, e.g., the results from the European-American MIRIAM project (Sandberg et al. 2011a), the Danish NordFoU project (Kragh 2010), as well as two Dutch studies (Roovers et al. 2005) and (Boere 2009).

In Sandberg et al. (2011a), the trailer measurements are a key component in creating a linear model for the rolling resistance's dependence on the road surface. The BRRC, BASt, and TUG trailers were used. The NordFoU project compared macrotexture in the form of mean profile depth (MPD) values with TUG trailer measurements with mixed results. The first Dutch study (Roovers et al. 2005) mentioned previously used the BASt and TUG trailers to determine differences in rolling resistance for different pavement types, but no low rolling resistance pavement was indentified within statistical significance. In Boere (2009) a good correlation between tire model predictions of rolling resistance and TUG trailer measurements was found. The tire-interaction model in Boere (2009) has two main components: One component accounts for rolling resistance of a smooth road, i.e., the hysterectic losses due to the flattening of the tire in the tire/surface contact zone. The second component relates to surface-texture-induced tire deformations and is based on Andersson and Kropp (2008), which uses a linear spring system and a nonlinear stiffness function to account for the tire-texture interaction.

A subproject of MIRIAM made an extensive comparative study between the BRRC, BASt, and TUG trailers on a test track in Nantes, France, and showed overall good correlations with both macrotexture and megatexture (Bergiers et al. 2011). Short-term repeatability was found to be acceptable with approximately 3% variation for the BRRC and BASt trailer and approximately 1% for the TUG trailer (Bergiers et al. 2011). Unfortunately,

(a) (b)

Fig. 1. (a) Rolling resistance trailer developed by the University of Gdansk, Poland (for further details on the device pictured, please refer to Wozniak et al. 2011a); (b) RoboTex laser profilometer for obtaining a three-dimensional surface profile (for further details on the device pictured, please refer to Rasmussen and Sohany 2011) (images courtesy of The Danish Road Directorate)

day-to-day variations were significant. As Bergiers et al. (2011) suggests, this should be subject to further study. More generally, this also shows that further research and perhaps trailer standardization are needed. Nevertheless, the trailer measurement approach seems fruitful: On one hand, many disturbing factors such as transmission losses and air resistance have been reduced or eliminated in contrast with coast-down experiments (see "Coast-Down Methods"), and on the other hand the trailers are still measuring rolling resistance on actual roads in contrast to the laboratory drum tests.

The above-mentioned trailers focus on the personal car, but recently BASt and Forschungsvereinningung Automobiltechnik (FAT) have developed a rolling resistance trailer using trucks and truck tires (Sandberg et al. 2011b, p. 66).

Coast-Down Methods

The coast-down method pinpoints all significant contributions to driving resistance, not merely the rolling resistance. The principle in coast-down measurement is to accelerate a vehicle to a certain speed and then let it roll freely in neutral gear or clutch down (Sandberg et al. 2011b). As the car "coasts down," velocity and time are measured as a minimum (Evans and Zemroch 1984), but other quantities like wind speed and road texture may be measured as well (Hammarström et al. 2009). The velocity is usually measured at a high frequency for accurate results. This method does not yield any direct results on rolling resistance, but must be fitted to a mathematical model by, e.g., estimating parameters with least-squares regression. The formulation and complexity of the model may vary depending on the experimental setting, sources of data, and so on. The development of models is treated in "Rolling Resistance Macromodeling."

Fuel Consumption Methods

Measurement of fuel consumption is the most general way of assessing rolling resistance because it includes all possible factors that influence the rolling resistance assessment. The tire rolling resistance obviously affects the fuel consumption (Schuring 1994; Hammarström et al. 2012), but because many factors influence the energy loss experienced by a car, it is difficult to pinpoint the rolling resistance loss in the fuel consumption method (Barrand and Bokar 2009). Modern fuel consumption models are complex and include many components such as, e.g., submodels of engine, powertrain, wheels, driver and brake control, road and meteorological conditions as detailed in, e.g., Sandberg (2001). The fuel consumption measurement method will not be discussed further in this paper; the reader is referred to Greenwood and Bennett (2001) for an introduction to fuel consumption measurement and modeling.

Surface Roughness and Texture Modeling

The basic challenge in roughness modeling is to extract useful information from road data. This depends on what kind of road data are available and what kind of information is sought. In the case of rolling resistance modeling, there are different kinds of information extractable on various length scales, as well as different measurement techniques. Fig. 2 shows effects related to vehicle and surroundings during driving, such as noise, rolling resistance, and tire wear, plotted against texture wavelength.

Two road measures, the International Roughness Index (IRI) and the MPD, are widely used in rolling resistance estimation today (Hammarström et al. 2009; Karlsson et al. 2011; Sandberg et al. 2011a, b; Kragh 2010); both have been derived from early measurement practices. They aim at modeling two of the texture types shown in Fig. 2, i.e., roughness and unevenness (IRI) and macrotexture (MPD). These two measures are briefly summarized now.

MPD has been derived from the sandpatch test, which was an early measure of macrotexture in the research of, e.g., skid resistance (Lupton and Williams 1972; Corley-Lay 1998). The test consists of spreading out a known amount of sand (or small glass spheres) on a road surface with a puck, in a large circle, and measuring the diameter [ASTM E965-96 (ASTM 2006), ISO 10844 (ISO 1994)]. The ratio between area covered and amount of sand used gives the mean texture depth (MTD) measure of macrotexture

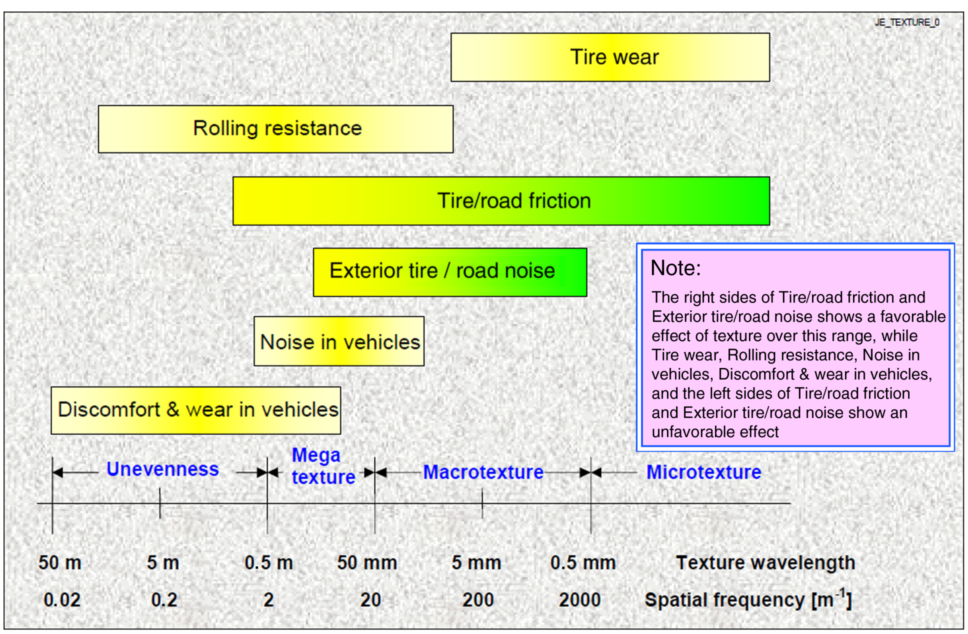

Fig. 2. Illustration of texture wavelengths, anticipated effects, and the classification into, e.g., megatexture, macrotexture; rolling resistance is affected by unevenness or roughness, megatexture, and macrotexture [reproduced with permission from Sandberg and Ejsmont (2002)]

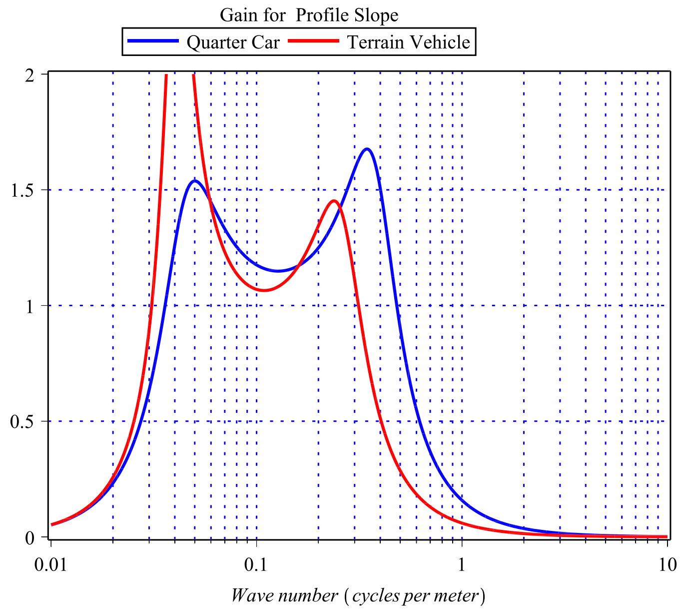

Fig. 3. IRI gain obtained from a quarter-car model with the golden car parameters (Sayers and Karamihas 1996) and the average parameters for terrain vehicle (Lajqi et al. 2012); the gain is highly sensitive to the parameters

as described in Annex A of ISO 10844 (ISO 1994). The sandpatch test is simple and robust, although prone to error because the test has to be carried out manually. With the advent of laser profilometers, MPD is used to describe macrotexture in a similar way, i.e., to obtain a number that correlates well with the sand patch test (Flintsch et al. 2003) and for which a simple transformation exists for conversion of MPD to MTD [ASTM E1845-09 (ASTM 2009), ISO 13473-1 (ISO 1997)]. In ISO 13473-1 (ISO 1997) the MPD is calculated as follows:

1. Take at least 10 100-mm laser profile segments of the road section where the MPD is to be found;
2. For each segment, substract the regression line such that the average vertical displacement is zero;
3. Find the maximum vertical displacement values of the first and second half of each segment and take the average of these; and
4. Take the average across all segments of the values found previously that yield the MPD value of the road section.

Both ISO and ASTM have developed several standards to account for macrotexture, and these texture measures are used today in road safety, maintenance, and research (Hall et al. 2009). In some cases the root-mean square of the road profile has been used instead of MPD (Boere 2009), although this is not common.

Measurement of road roughness has been developed since the 1920s according to Sayers and Karamihas (1998, p. 39), and road roughness has been measured by the so-called response-type road roughness measuring systems (RTRRMSs). The general construction of RTRRMSs consists of a wheel mounted to a spring that records and accumulates any bumps in the longitudinal road profile (measured in, e.g., meters). By dividing this quantity with the distance travelled (in, e.g., kilometers), a measure of road roughness (m/km) is obtained. Other roughness measurement techniques have been developed, e.g., the rod and level profiler and the inertial profiler (Sayers and Karamihas 1998, 1996; Visser 1982; Bester 1984; Hveem 1960), but the RTRRMS devices form the basis of the IRI measure. They were developed as a common standard for calibration (Gillespie et al. 1980; Sayers et al. 1986; Bennett 1996)

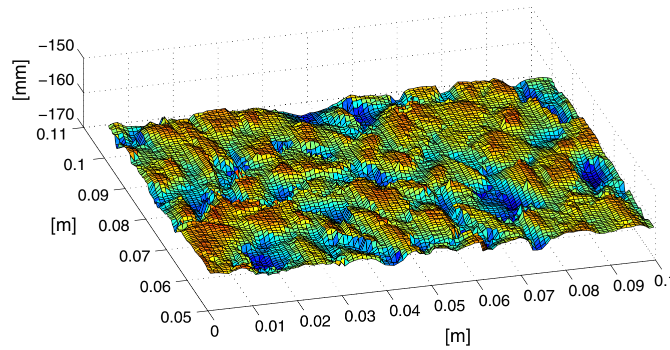

Fig. 4. Detailed surface elevation of the Værløse, Denmark, airfield pavement as obtained by the RoboTex laser profilometer shown in Fig. 1(b)

of RTRRMSs and are based on a quarter-car model simulating the RTRRMS measuring device. The IRI measure stems from a mathematical model that represents a (quarter) vehicle's damping response to the road's longitudinal profile. An equivalent way of describing this is in terms of a frequency response function of the road profile frequency spectrum, as shown in Fig. 3. Modifications to the IRI measure have been proposed in, e.g., Sayers (1989), where the quarter-car model is expanded to a half-car model.

Instead of just modeling and measuring a fraction of the frequency domain like macrotexture or road unevenness and roughness, the entire profile spectrum can be used. This simple but powerful approach to road texture and roughness modeling transforms the longitudinal road profile into the frequency domain. This has been especially useful with the introduction of laser profiles because they measure with high frequency and precision, c.f. the profile shown in Fig. 4. A standardization of the profile spectrum can be found in ISO 13473-4 (ISO 2008). The disadvantage is that it does not yield a single number, but, e.g., correlation analyses working in the frequency domain seems ideal as shown in Sandberg (1990). The spectral analysis approach has also been used to highlight the influence of megatexture on rolling resistance (and noise), which has usually been attributed to macrotexture (Descornet 1989, 1990).

Different texture types are defined in terms of wavelengths (Fig. 2). This approach may hardly deserve the term modeling, but by reducing a laser profile to a spectrum, an idealization is made and information is lost. More specifically, the surface profile spectrum omits phase information, and by doing this, surface properties relevant to rolling resistance modeling are lost. In Männel and Beckenbauer (2007), a discussion of these matters is undertaken with examples of different schematic profiles that should result in different tire-surface interaction dynamics, but which yields similar power spectra. Another example of different profiles with characteristics invisible in surface spectra are given in Pinnington (2012).

Recently, various other approaches have been investigated as alternatives to the classical measures mentioned previously. In Anfosso-Lédée and Do (2002), certain geometric descriptors and their properties have been developed and extracted from laser profiles. More specifically, peaks, valleys, and the angles of these are calculated from the profiles and correlated with the tire-road noise (Anfosso-Lédée and Do 2002). Even though a correlation was observed, it was concluded that further research is needed (Anfosso-Lédée and Do 2002, p. 167), and the methods' usefulness has been disputed (Sandberg and Ejsmont 2002). Another approach is the use of fractals in surface modeling (Panagouli and Kokkalis 1998; Kokkalis et al. 2002) and design (Yeggoni et al. 1996).

By estimating the fractal dimension of a surface laser profile, a surface measure relating to macrotexture and microtexture is obtained. This has been shown to correlate well with the skid resistance number (SN) (Panagouli and Kokkalis 1998; Kokkalis et al. 2002), and in addition the fractal dimension drops with SN, as expected, when the pavement wears (Kokkalis et al. 2002). Fractals are also being used in Pinnington (2012) in which a surface model is constructed and comprises three different layers corresponding to different length scales. Yet another approach is to use classical measures like MPD, obtained from road laser profiles, in conjunction with an envelope algorithm that mimics the viscoelastic properties of the tire. A purely empirical formula developed in Meier et al. (1992) has increased the correlation between MPD and trailer measurements of rolling resistance in studies from the MIRIAM project (Sandberg et al. 2011a). Similar enveloping methods have been reported and developed in Klein and Hamet (2004), based on the viscoelastic properties of the tire instead of a purely empirical algorithm. In addition, Klein and Hamet (2004) discuss how the enveloping procedure affects the surface profile spectra.

Contact Models

In the 1980s efforts to understand and quantify texture effects on the tire-pavement interaction were limited. There were many difficulties in theoretically and experimentally determining the many individual contact areas and contact pressures produced by irregularly shaped asperities indenting the tire tread. In Yong et al. (1980), an analytical model is developed using experimental data for individual tire types to predict the contact area. A numerical method is demonstrated in Yong et al. (1980) to approximate the collective contact stress for individual contact areas using a computational algorithm requiring only the two-dimensional (2D) road profile geometry and tire inflation pressure as input.

Gall et al. (1993) introduced a finite-element model for the tire in the contact area, focusing on the correct representation of the contact area including the edge effects of the tire-soil contact, a friction law including normal stress, and correct modeling techniques such as the use of symmetry.

By the beginning of this century, considerable advances had been made in numerical computing resources, giving the opportunity to investigate the contact area in more detail. It became possible to include the nonlinear behavior of the contact zone that was previously overlooked or simplified. In Andersson and Kropp (2008) the contact geometry is discretized into smaller length scales using multiple pairs of matching points with nonlinear springs between each pair of contact elements. The stiffness functions of these springs are determined from a method for assessing the stiffness of the nonlinear springs based on detailed scans of the surface geometry, elastic data of the tread, and a flat circular punch-indenting method for normal (out-of-plane) contact model of an elastic layer. The Newton-Raphson iterative scheme is used to solve the nonlinear contact equations. Green's functions calculate analytically the dynamic response of the tire by convolving the contact forces.

Ivanov et al. (2010) identify and address parameters that characterize the interaction of a tire-soil interaction using fuzzy set theory. The contact of the tire with both hard and soft soil is discussed with specific focus on how to handle the parameters of tire-soil friction and rolling resistance. The advantage of these methods lies in their ability to take into account fluctuating external conditions that are not directly related to a vehicle.

Dubois et al. (2011) deal with the numerical study of a frictionless viscoelastic tire-road contact area. This is done by means of a macroscale approach in which only the contact forces are calculated for a rough multi-indentation surface of a viscoelastic half-space based on an imposed load at road surface asperities peaks. This approach takes into account both the viscoelastic behavior of the tire and the roughness of the road surface. The viscoelastic solution is reduced to an elastic solution, significantly reducing the calculation time, and a simplified description of the viscoelastic material behavior by a rheological model is used.

The Lund-Grenoble (LuGre) model, describing three-dimensional tire friction dynamics simplified by assuming constant slip along the contact patch, is elaborated in Deur et al. (2005). This model includes the effects of lateral deformation of the tire tread, which leads to varying slip speeds along the contact patch. This is done using a stepwise approximation of the slip speed. In Faraji et al. (2010), this simplification is not used and a quarter-car model and an average lumped LuGre model are used instead.

The current trend in modeling the contact zone between tire and pavement is to include all the major aspects, i.e. noise, rolling resistance, and skid resistance, cf. Andersson et al. (2012).

Tire Models

The relation of tire rolling resistance to the viscoelastic and dynamic hysteresis properties of typical tire materials is complex. The combination of operational variables such as pressure, load, speed, and deflection, and the tire design variables complicate the characterization of the stress-strain hysteresis contribution of each tire component and the interaction between them, and hence the determination of the rolling resistance of the tire on a given surface. In the 1980s, Williams and Dudek (1983) compared the sinusoidal radial load-deflection cycling of a rolling tire with a nonrolling tire. From these comparisons, relations were made between the footprint load-deflection hysteresis and the rolling resistance drag force, and the contribution of tread and sidewall deformation to the hysteresis was determined.

An alternative to viscoelastic models is given in Luchini et al. (1994), detailing a finite-element strain-based model using directional incremental hysteresis to predict rolling resistance. The tire material model is here developed for the rubber components only, while the cords are included for structural aspects of the model. Shida et al. (1999) presents a static finite-element model for fiber-reinforced rubber capable of handling anisotropic loss factors. The algorithm proposed by Shida et al. (1999) estimates the energy dissipation from the hysterectic loss in a tire, using the variations of the approximated stresses and strains. These stresses and strains are calculated using a Fourier series with a viscoelastic phase lag in the frequency domain.

Due to hysteresis losses, heat energy is generated, which leads to higher tire pressure and thus lower rolling resistance and vehicle fuel consumption. The complex relationship between the various design attributes and operating conditions makes it difficult to develop analytical models. Various attempts have been made to model the total behavior through a semicoupled representation. Three models, the dissipation, deformation, and thermal models, have been considered. In the late 1990s, Park et al. (1997) used these three major analysis models and viscoelastic theory to calculate the heating of a rolling tire. Results were compared with physical measurements and comparisons made between quadratic and linear finite elements. Due to the delicate nature of the prediction of energy loss in a tire, specifically the numerical analysis of the strain dependent carbon black-filled rubber, special attention is paid to the material representation in Ebbott et al. (1999). In this paper the dissipation and deformation models are based on the strain-amplitude dependence of carbon black-filled rubber, and the thermal model does not require the use of correlation coefficients for accurate results. The algorithm developed takes both strain and temperature dependence into account. Tire temperatures are obtained by solving steady-state linear heat transfer equations using the finite-element technique. This algorithm was used and expanded in the following decade in Narasimha Rao et al. (2006). A three-stage finite-element model consisting of a deformation model, a dissipation model, and a thermal model is used to determine characteristics for tires with smooth and circumferential groove tread patterns. In Narasimha Rao et al. (2006) variations in several aspects affecting rolling resistance is made and the results discussed. These include the tire rolling speed, tread profile, inflation pressure, a varying normal load, and ambient temperature. Comparisons are made between a flat road surface and a circular drum. Various hyperelastic and viscoelastic properties of the tread material are considered. The results are summarized in a table showing that the effective rolling radius is insensitive to parameter variations, whereas the rolling resistance (and hence the total energy loss per revolution) is insensitive to rolling speed, convection loss, and friction, but increases significantly with increasing normal load, tread profile, and tread material loss modulus and decreases significantly with increasing ambient temperature, convection loss, tread mechanical stiffness, and tread thickness.

A widely used empirical tire model is expressed in the so-called Magic Formula, the development of which started in Delft, Netherlands, in the mid-1980s (Pacejka 2012). By way of example the authors give the relation between the side force F_y and the slip angle α, i.e., the angle between the lateral and forward velocities of the wheel center

$$F_y = D\sin(C\arctan\{B\alpha - E[B\alpha - \arctan(B\alpha)]\}) \quad (1)$$

Here B, C, D, and E are parameters that are determined by fitting the relation to data. The Magic Formula produces characteristics that closely match measured curves for the side force and longitudinal force as a function of their respective slip qualities. A typical

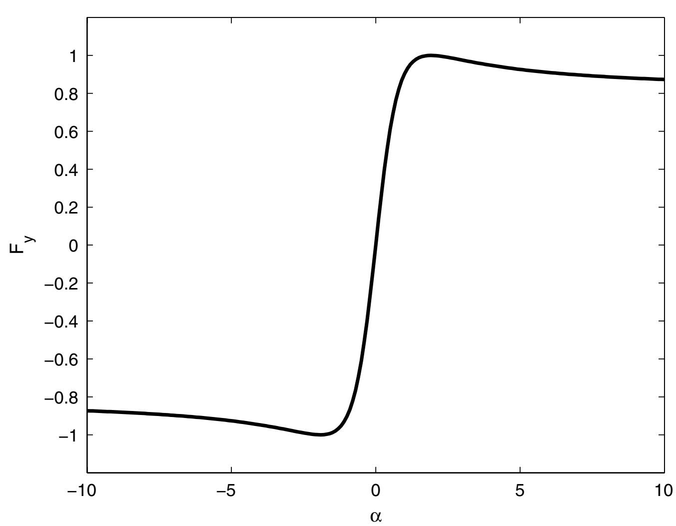

Fig. 5. Illustration of the Magic Formula (Pacejka 2012) showing the nondimensionalized side force F_y ($D = 1$) as a function of the slip angle α ($B = 1$), with parameters $C = 1.4$ and $E = -0.2$

graph of the magic formula is shown in Fig. 5. For a full treatment of the Magic Formula, the reader is referred to Pacejka (2012). The Magic Formula has been extended to cope with large camber angles and tire inflation pressure by Besselink et al. (2010). The ability to deal with pressure changes eliminates the need to have separate parameter sets for different tire pressures, leading to a reduction in the total number of measurements required. In addition, the description of the rolling resistance and overturning moment is improved. Changes in the modeling of the tire dynamics allow a smooth and consistent switch from simple first-order relaxation behavior to rigid ring dynamics. The effect of inflation pressure on the loaded radius and the tire enveloping properties is discussed by Besselink et al. (2010) and some results are given to demonstrate the abilities of the model.

The development of tire models is constantly improving and expanding. A methodology using probabilistic characteristics of a vehicle and road to model the interaction between them, including rolling resistance, is presented in Vantsevich and Stuart (2008). The authors represent the interaction of the vehicle with the road by means of a quarter-car model, the characteristics of which are varied randomly for the interaction with the vehicle surroundings. A full two-dimensional semianalytical model for viscoelastic cylinders rolling on a rigid surface is developed in Qiu (2009). Problems arising from high-speed contact for layered viscoelastic rollers rolling on a rigid surface and standing-wave phenomena are addressed here.

The previously mentioned papers all use a finite-element model for simulation of various aspects of the rolling pneumatic tire. A general review of the literature on finite-element modeling of rolling tires is given by Ghoreishy (2008). This review gives a survey on finite-element modeling of rolling tires, application of rolling tire models, and finite-element codes. The challenge is to obtain realistic material models, model the tread blocks, further develop the finite-element models to include multiphysics, include transient behavior, and finally include the modeling of nonrigid surfaces. Finite-element modeling of the rolling tire is complex, and although more comprehensive and true to physical first principles, for many applications they are not yet fast enough for realistic vehicle simulations. Here empirical models are still needed.

Rolling Resistance Macromodeling

Apart from the detailed tire and contact models of rolling resistance, more empirical macromodels exist and have been in development since 1935 according to Petrushov (1997). These models focus on coast-down experiment data (see experiment description in "Rolling Resistance Measuring Techniques"). Initially, the primary goal of these models was to assess vehicle aerodynamic drag (White and Korst 1972; Walston et al. 1976; Buckley et al. 1976), which correlates well with wind-tunnel experiments (Eaker 1988; Buchheim et al. 1980; Bester 1984; Swift 1991; Korst and White 1990). In recent years, coast-down models have been used for rolling resistance assessment as well (Roussillon 1981; Hammarström et al. 2009; Karlsson et al. 2011). The general formulation of coast-down models is based on Newton's second law (Hammarström et al. 2009, p. 24), i.e.

$$F_{\text{total}} = m\frac{dv(t)}{dt} = F_{\text{roll}} + F_{\text{air}} + F_g + F_{\text{misc}} \quad (2)$$

where the total force F_{total} acting on the coasting vehicle is given by the rolling resistance contribution F_{roll}, aerodynamic drag F_{air}, the gravity's component in the direction of motion F_g, and F_{misc} representing various other forces like side force or transmissionlosses, although the latter can be included in F_{roll} (Hammarström et al. 2009; Karlsson et al. 2011). The mathematical formulations of the coast-down model equations, the experimental setup, and quantities measured vary greatly from study to study in the literature. A few of the different approaches are summarized below.

The complexity of the mathematical models varies from relatively simple equations governing vehicle motion with only velocity and gradient data and a simple empirical quadratic reistance model (Evans and Zemroch 1984) like

$$\frac{dv(t)}{dt} = a_0 + a_1 v(t) + a_2 v(t)^2 \quad (3)$$

to much more complex nonlinear models (Hammarström et al. 2009; Karlsson et al. 2011). The complex models depend on a large amount of additional data being measured, such as gradient, road crossfall, macrotexture, roughness, and meteorological conditions, which greatly enhances the resulting rolling resistance estimates. The advantages of choosing simple models lies in the possibillity of obtaining an analytical solution to the differential equation model as in, e.g., Ivens (1987), thus greatly reducing the computational demands. In Petrushov (1997), the analytical solution is used to convert the velocity-time function to distance-time instead, thus reducing error sources. Another approach is to simplify the experimental setting by, e.g., having a flat test section such that the road gradient can be neglected (Hamabe et al. 1985; Djordjevic et al. 2009) or using data from an anemometer mounted on the vehicle (Buckley 1995). As mentioned previously, coast-down models have been used for several purposes, and apart from rolling resistance and aerodynamic drag assessment, the methodology has been used to estimate fuel consumption (Hunt et al. 2011), transmission losses (Dunn et al. 2009), and maximum vehicle speed (Lieh 2008).

Although macromodels of rolling resistance are primarily based on coast-down models, other approaches have also been developed like, e.g., viscoelastic models used in connection with laboratory rolling resistance experiments to predict rolling resistance of tires. These experiments have shown that tire rolling resistance energy loss is correlated with hysteresis loss in the tire (Pillai 1995; Pillai and Fielding-Russell 1991). Thermomechanical models have also been developed and used to predict transient rolling resistance (Mars and Luchini 1999), and thermomechanical principles were essential in developing a new macromodel of rolling resistance that showed the importance of tire temperature on rolling resistance (Sandberg 2001).

Concluding Remarks

This paper briefly reviewed the state of the art of rolling resistance modeling. Regarding the optimal quantitative characterization of a road surface for predicting the rolling resistance, more work is needed. On the one hand, MPD, although a purely empirical adaptation of the sand patch test derived MTD to laser profiles, is widely used throughout the rolling resistance literature. MPD's popularity is probably due to the historical background and the simplicity of the algorithm. This combined with correlations with diverse rolling resistance measurements makes it a practical choice when a texture measure needs to be extracted from laser profile data. On the other hand, in recent studies such as, e.g., Sandberg et al. (2011a), MPD was combined with a physically intuitive envelope procedure that improved correlations substantially. Taking fundamental physical considerations into account when using laser profiles or other modern measurement techniques for surface characterization thus seems promising. A similar trend can be seen in the development of macromodels, from the simple and purely empirical approach of coast-down modeling in Evans and Zemroch (1984) to elaborate models based on physical principles in Karlsson et al. (2011).

Because of advances in numerical computing resources, it is now possible to model the tire pavement contact zone in much more detail than previously. This development will continue and likely be combined with more detailed tire models. Tire modeling depends on the overall purpose of modeling. Magic Formula type of modeling is used for fast response in real-time vehicle modeling, whereas the tendency in modeling the interaction between tire and pavement surface is to use some kind of finite-element modeling.

In a recent study (Nielsen and de Fine Skibsted 2010), it has been estimated that potential savings in fuel consumption (and hence CO_2 emissions) from optimizing the pavement with respect to rolling resistance represent a value to society as large as the entire cost of maintaining the pavement. Because the optimization in an asset management system requiring reliable models of rolling resistance, further research in rolling resistance modeling is warranted by their benefits to society.

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